DP1 Complex Numbers

Summative – DP1 Math HL

Complex Numbers

Your Name:

DP1 – Mathematics HL – Complex numbers.

23rd February. 2017.

Solve any 10 questions on the answer sheets.

Question 1: [6]

Simplify and write the following in the form of $a + b\boldsymbol{i}$**:** [No calculator please]

(a) $\frac{1}{i}\left(\frac{1}{\sqrt{5}} - \frac{3i}{\sqrt{3}}\right)$

(b) $\frac{1}{1+i}$

(c) $\frac{1}{1-i}$

(d) $2i - \frac{2}{i+\sqrt{3}}$

(e) 3^{i}

(f) $(-3 - 3\mathrm{i})^2$

Question 2: **[6]**

Let $z = 1 + i\sqrt{3}$ and $w = 3\sqrt{3} - 3i.$

(a) Find the modulus and the argument of *z* and *w*.

(b) Represent *z* and *w* on the same Argand diagram.

(c) Find the modulus and the argument of the product $\boldsymbol{zw}$. Comment on your answer.

Question 3: **[6]**

Consider the complex numbers *z* = 1 + 2i and *w* = 2 +*a*i, where $a \in \mathbb{R}$.

Find *a* when

(a) $|w| = 2|z|$;

(b) Re (*zw*) = 2 Im(*zw*).

Question 4: **[6]**

If $z = \text{cis}(\theta)$, prove that: $i\tan(\theta) = \frac{z^2-1}{z^2+1}$

Question 5: **[6]**

Using de Moivre's theorem, or otherwise, prove that:

(a) $\sin 3x = 3\sin x - 4\sin^3 x$

(b) $\cos 3x = 4\cos^3 x - 3\cos x$

Question 6: **[6]**

Find the fourth roots of unity in the form: $r\ \text{cis}(\theta)$. [Hint: solve $z^4 = 1$]

Question 7: **[6]**

Show that:

(a) $(n-1)(n^2+n+1) = n^3 - 1$

(b) Given: $z = e^{i(2\pi/3)}$, Show that $z^3 = 1$, and $1 + z + z^2 = 0$

(c) Express each of the following expression in terms of z. (write in simplest form)

i. z^8

ii. $(1-z)^2 + 4z$

iii. z^{100}

Question 8: **[6]**

Show that $\cos z = \frac{e^{iz}+e^{-iz}}{2}$. Hence find the exact value of $\cos i$. Also define an expression for $\sin z$, where z is a complex number.

Question 9: **[6]**

Given $z = \cos\theta + \text{i}\sin\theta$, show that: $z^n - \frac{1}{z^n} = 2\text{i}\sin(n\theta)$.

Question 10: **[6]**

Find, in its simplest form, the argument of $(\sin x + i(1-\cos x))^2$ where $\boldsymbol{x}$ is an acute angle.

Question 11: **[6]**

(a) Find b where: $\frac{2+b\text{i}}{1-b\text{i}} = \frac{7}{10} + \frac{9}{10}\text{i}$

(b) Given that $(a+b\text{i})^2 = 3 + 4\text{i}$ obtain a pair of simultaneous equations involving a and b. Hence solve for a and b.